SOLUTION (K.V.S.-P.G.T. 2016)

101 (1)

$\bar{a} = i - 2j + k \qquad \bar{b} = 4i - 4j + 7k$

Projection of $\bar{a}$ on $\bar{b}$ $= \dfrac{\bar{a}\cdot\bar{b}}{|\bar{b}|}$

$= \dfrac{4+8+7}{\sqrt{16+16+49}}$

$= \dfrac{19}{\sqrt{81}} = \boxed{\dfrac{19}{9}}$ Ans

option - (b)

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102 $\alpha = \beta = 60^\circ$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\cos^2 60 + \cos^2 60 + \cos^2\gamma = 1$

$\frac{1}{4} + \frac{1}{4} + \cos^2\gamma = 1$

$\cos^2\gamma = 1 - \frac{1}{2}$

$\cos\gamma = \frac{1}{\sqrt{2}}$

$\boxed{\gamma = 45^\circ}$ Ans option - (a)

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103 $\bar{a}\cdot(\bar{b}+\bar{c}) + \bar{b}\cdot(\bar{c}+\bar{a}) + \bar{c}\cdot(\bar{a}+\bar{b}) = 0$

$2(\bar{a}\cdot\bar{b} + \bar{b}\cdot\bar{c} + \bar{c}\cdot\bar{a}) = 0$

$|\bar{a}+\bar{b}+\bar{c}|^2 = a^2 + b^2 + c^2 + 2\cdot(\bar{a}\cdot\bar{b} + \bar{b}\cdot\bar{c} + \bar{c}\cdot\bar{a})$

$= 1 + 16 + 64$

$= 81$

$\boxed{|\bar{a}+\bar{b}+\bar{c}| = 9}$ option - (c)

B

(104) Sol^n

$$a_2 - a_1 = (2-1)\hat{i} + (4-2)\hat{j} + (5-3)\hat{k}$$

$$= \hat{i} + 2\hat{j} + 2\hat{k}$$

$$b_1 \times b_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{vmatrix}$$

$$= \hat{i}(15-16) - \hat{j}(10-12) + \hat{k}(8-9)$$

$$= -\hat{i} + 2\hat{j} - \hat{k}$$

$$S \cdot D = \frac{(a_2 - a_1)\cdot(b_1 \times b_2)}{|b_1 \times b_2|} = \frac{-1+4-2}{\sqrt{1+4+1}} = \boxed{\frac{1}{\sqrt{6}}}$$

option - (b)

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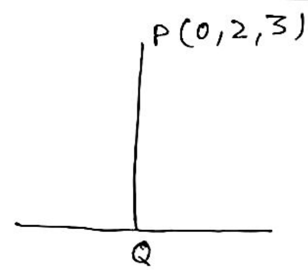
(105) Sol^n

The standard deviation of the first n natural number is =

$$= \sqrt{\frac{n^2-1}{12}}$$

---

(106) Sol^n



$$\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3} = t$$

by option Foot of Perpendicular satisfy the ~~the~~ eq^n of line

Here option (c) satisfy the eq^n of line

$$\frac{2+3}{5} = \frac{3-1}{2} = \frac{-1+4}{3} = t$$

$$1 = 1 = 1 = t$$

Ans option (c)

(107) Sol^n

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1+n_2}$$

$$= \frac{3\times14 + 2\times18}{5} = \frac{42+36}{5} = \frac{78}{5} = \boxed{15.6} \text{ Ans}$$

Option (d)

---

(108) Sol^n

The G.M of $(a\,l)^{1/2}$

$$= (3\times243)^{1/2} = (729)^{1/2}$$

$$= \boxed{27} \text{ Ans}$$

Option (b)

---

(109) Sol^n

Mean $\bar{x} = \frac{1+2+3+4+5+6}{6} = \frac{21}{6} = \frac{7}{2}$

$$S.D = \sqrt{\frac{n^2-1}{12}} = \sqrt{\frac{6^2-1}{12}} = \sqrt{\frac{35}{12}}$$

$$\boxed{\text{Mean} = \frac{7}{2},\quad S.D = \sqrt{\frac{35}{12}}} \text{ Ans}$$

Option (a)

(110) Sol^n

$$S = \sqrt{\frac{\sum_{i=1}^{100}(x_i-\bar{x})^2}{100}} \Rightarrow \sqrt{\frac{\sum_{i=1}^{100}(x_i-50)^2}{100}} = 5 \Rightarrow \sum_{i=1}^{100}(x_i-50)^2 = 25\times100$$

$$\Rightarrow (x_1-50)^2+(x_2-50)^2+\cdots\cdots(x_{100}-50)^2 = 2500$$

$$(x_1^2+x_2^2+x_3^2+\cdots x_{100}^2)+(50^2+50^2+50^2+\cdots\cdots 100\text{ time}) - 2\times50(x_1+x_2+\cdots\cdots x_{100}) = 2500$$

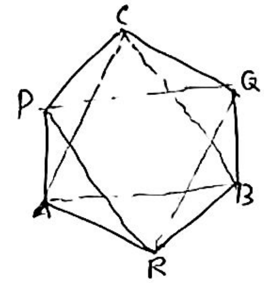
$$(x_1^2+x_2^2+x_3^2+\cdots x_{100}^2) + 2500\times100 - 2\times50(50\times100) = 2500$$

$$\sum_{i=1}^{100} x_i^2 + 250000 - 500000 = 2500$$

$$\sum_{i=1}^{100} x_i^2 = 250000+2500 = 252500 \text{ Ans} \quad \text{Option-(c)}$$

9

(111) soln There are two equiletral Δ PQR and Δ ABC



$$S.S = {}^6C_3 = \frac{6\times5\times4}{3\times2} = 20$$

$$F.S = 2$$

$$P(A) = \frac{F.S}{S.S} = \frac{2}{20} = \boxed{\frac{1}{10}}$$

option - (d)

---

(112) $P(C) = x$, $P(B) = x/2$, $P(A) = x/3$

$$P(A) + P(B) + P(A) = 1$$

$$\frac{x}{3} + \frac{x}{2} + x = 1$$

$$= 2x + 3x + 6x = 6$$

$$11x = 6$$

$$x = \frac{6}{11}$$

$$P(A) = \frac{6}{11\times3} = \boxed{\frac{2}{11}}$$

option - (b)

---

(113) soln

Total No. of three digits = $4\times5\times5$

No. of three digit of same digits = 4

required prob. = $\frac{4}{4\times5\times5} = \frac{1}{25}$

$$\boxed{P(A) = \frac{1}{25}}$$

option - (d)

(114) Sol^n

SOLUTION KVS PGT 2016

At least one of event A and B occurs $P(A \cup B) = 0.6$

And A and B occur simultenusly $P(A \cap B) = 0.2$

$$P(A \cup B) + P(A \cap B) = P(A) + P(B)$$

$$= 1 - P(\bar{A}) + 1 - P(\bar{B})$$

$$0.6 + 0.2 = 2 - P(\bar{A}) - P(\bar{B})$$

$$P(\bar{A}) + P(\bar{B}) = 2 - 0.8$$

$$\boxed{P(\bar{A}) + P(\bar{B}) = 1.2}$$ Ans

option - (c)

---

(115) Sol^n

for linearly Independent

$L.I \neq 0 \quad |A| \neq 0$

for linearly dependent $|A| = 0$

$$\begin{vmatrix} 2 & 2 & 1 \\ 0 & 2 & -1 \\ 0 & 2 & 1 \end{vmatrix} = 2(2+2) = 8$$

$|A| \neq 0$

$\Rightarrow$ linealy Independent

option (E) Ans

---

(116) Sol^n PROBABILITY

$S.S = 11$

$F.S = 3 \quad (a, i, o)$

required prob $\boxed{P(A) = \frac{F.S}{S.S} = \frac{3}{11}}$ Ans

option (4)

(117)

$P(t) = a_0 + a_1 t + a_2 t^2$

where $a_i \in R$, $i = 0, 1, 2$

check by one-by one option, we find out option (2) $1-4t+t^2$, $-2+t-t^2$, $-7t+t^2$

form different basis for $P_2(t)$

Option - (b)

Ans

---

(118) Solⁿ

$\bar{x} = -15\hat{i} + 3\hat{j} - 9\hat{k}$, $\bar{y} = 15\hat{i} + 9\hat{j} + k \cdot \hat{k}$

For orthogonal $\bar{x} \cdot \bar{y} = 0$

$= -225 + 27 - 9k = 0$

$9k = -198$

$$\boxed{k = \frac{-198}{9} = -22}$$ Ans

option - (b)

---

(119) Solⁿ

$$\alpha \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + \nu \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 26 \\ 12 \\ 2 \end{pmatrix}$$

$\alpha + \beta = 26$

$-\alpha + \beta = 12$ — $\beta = 19$

$-\alpha + \nu = 2$ then $\alpha = 7$

and $\nu = 9$

$$\boxed{(\alpha, \beta, \nu) = (7, 19, 9)}$$ Ans

option - (d)

SOLUTION KVS PGT 2016

(120) Sol^n –

$n(C) = 20$, $n(H) = 15$, $n(C \cap H) = 5$

$n(C \cup H) = n(C) + n(H) - n(C \cap H)$

$= 20 + 15 - 5 = 30$

$n(\overline{C \cup H}) = 50 - n(C \cup H)$

$= 50 - 30 = 20$

$\boxed{n(\overline{C \cup H}) = 20}$ Ans option – (b)

---

(121) Sol^n

$X = \{ 8^n - 7n - 1 \mid n \in N \}$ And

$Y = \{ 49n - 49 \mid n \in N \}$ then

$X = \{0, 49, 490 ------ \}$

$Y = \{0, 49, 98 -------- \}$

$\Rightarrow$ X is subset of Y

$\boxed{X \subset Y}$ Ans option (a)

---

(122) Sol^n

$2^m = 2^n + 112$

$2^m - 2^n = 112$

$2^n(2^{m-n} - 1) = 112$

$= 2^4 \times 7$

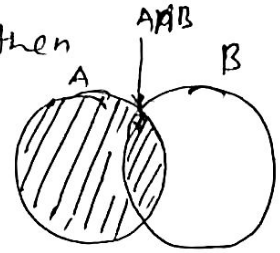
$\Rightarrow \boxed{n = 4 \quad \text{and} \quad m = 7}$

$(m, n) = (7, 4)$ Ans

Option – (c)

(123) Solⁿ

if A and B are two sets then

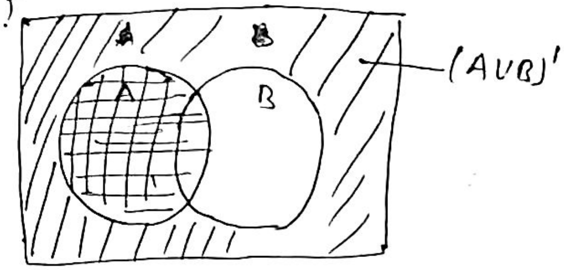


$$\boxed{A \cup (A \cap B) = A}$$

Option – (d) Ans

---

(124) Solⁿ If A and B are two sets and A' denotes the complement of A, then $A \cap (A \cup B)' = ?$



$$= \boxed{A \cap (A \cup B)' = \phi}$$

Option – (c) Ans

---

(125) Solⁿ

Range of $= \dfrac{1}{1 - 2\cos x}$

$-1 \le \cos x < 1$

$-2 \le 2\cos x < 2$

$-2 \le -2\cos x < 2$

$-2 + 1 \le 1 - 2\cos x \le 2 + 1$

$-1 \le 1 - 2\cos x \le 3$

$$\boxed{-1 \le \frac{1}{1 - 2\cos x} \le \frac{1}{3}}$$

$\left[-1, \frac{1}{3}\right]$ Ans

Option – (b)

---

(126)

$F(x) = \sqrt{1 + x^2}$, $F(y) = \sqrt{1 + y^2}$

$f(xy) = \sqrt{1 + x^2 y^2}$

$F(x) \cdot F(y) = \sqrt{1 + x^2 + y^2 + x^2 y^2}$

$$\boxed{F(x, y) \le F(x) \cdot F(y)}$$

Ans

Option – (d)

(127) Solⁿ

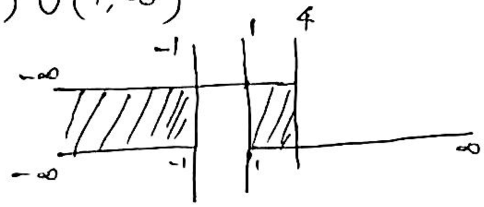
$$F(x) = \sqrt{4-x} + \frac{1}{\sqrt{x^2-1}}$$

for domain

$4 - x \geq 0$ and $x^2 - 1 > 0$

$x \leq 4$ $\quad$ $x \in (-\infty\ -1) \cup (1, \infty)$

$x \in (-\infty, 4)$



Now domain

$$\boxed{x \in (-\infty\ -1) \cup (1, 4]}$$

option - (a)

---

(128)

$f(x) = g(x)$

$3x^2 - 1 = 3 + x$

$3x^2 - x - 4 = 0$

$x(3x-4) + 1(3x-4) = 0$

$(x+1)(3x-4) = 0 \Rightarrow \boxed{x \in \{-1, \frac{4}{3}\}}$ Ans

Option- (a)

---

(129) Solⁿ

$F(x) = px + q$

$F(-1) = -p + q = -5$, $F(3) = 3p - q = 3$

$p - q = 5$ —(i) $\qquad$ $3p - q = 3$ —(ii)

from (i) and (ii)

we get $\boxed{p = 2,\ q = -3}$

option - (b) Ans

---

(130) Solⁿ

$x^n - 1$ is devisible by $x - k$ then least value of $k = 1$

$\Rightarrow x^n - 1$ is devisible by $x - 1$

$\boxed{k = 1}$ Ans

option (c) a

1 (131) Soln

$3(5^{2n+1}) + 2^{3n+1}$ is devisible by

for $n=1$ $\Rightarrow$ $3(5^3) + 2^4$

$\Rightarrow$ $375 + 16 = 391$

391 is devisible by $\underline{\underline{17}}$ Ans

so option (b) Ans

---

(132) The Number of possible outcomes when a coin is tossed 6 times is $= 2^6$

$= \underline{\underline{64}}$ Ans

option - (d) Ans

---

(133)

If total No. of Hand-shakes is $= 66$

then total No. of persons in room is =

$^nC_2 = 66$

$n(n-1) = 132$

$\boxed{n = 12}$ Ans

Option - (b)

---

(134)

Total No. of Nine digit

$= 9 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$= \underline{\underline{9 \times \lfloor 9}}$

Ans

option - (d)

(135) Solⁿ

If. ${}^nC_{12} = {}^nC_8$

$\Rightarrow n = 12 + 8 = 20$

$\boxed{n=20}$ option - (a)

---

(136) Solⁿ

$(z+3)(\bar{z}+3) = ?$

we know $z\bar{z} = |z|^2$

$(z+3)(\overline{z+3}) = |z+3|^2$

$(z+3)(\bar{z}+3) = |z+3|^2$ option - (a)

---

(137) $\sin\theta + i\cos 2\theta = \cos\theta - i\sin 2\theta$

$(\sin\theta - \cos\theta) + i(\cos 2\theta + \sin 2\theta) = 0 + 0i$

$\sin\theta - \cos\theta = 0$ and $\cos 2\theta + \sin 2\theta = 0$

$\tan\theta = 1$ $\quad$ $\tan 2\theta = -1$

$\theta = \pi/4$ $\quad$ $\theta = \frac{3\pi}{4}$

so No value of θ

option - (d)

---

(138) $\left(\frac{1+i}{1-i}\right)^x = 1 \Rightarrow \left[\frac{1+i}{1-i} \times \frac{1+i}{1+i}\right]^x = 1$

$= \left[\frac{1-1+2i}{1+1}\right]^x = 1 \Rightarrow [i]^x = 1$

$\Rightarrow \boxed{x = 4n}$ Ans

Option - (b)

(139) - sol^n

Let $x, y \in \mathbb{R}$

$x + iy$ is non-real complex number then

$\boxed{y \neq 0}$ Ans

option - (d)

---

(140) sol^n if $|x| > a$

$\Rightarrow x > a$ and $x < -a$

$\boxed{x \in (-\infty, -a) \cup (a, \infty)}$

option - (d)

---

(141) if $|x-1| > 5$

$\Rightarrow (x-1) > 5$ and $x-1 < -5$

$x > 6$ and $x < -4$

$\boxed{x \in (-\infty, -4) \cup (6, \infty)}$ option (c)

---

(142) x, y and a are Real Number where $x < y$, $a < 0$

Let $2 < 3$

$\cancel{\frac{1}{2} > \frac{1}{3}} \Rightarrow \cancel{\frac{1}{x} > \frac{1}{y}}$ $x < y$

$\boxed{\frac{x}{a} > \frac{y}{a}}$ - (c)

$\frac{\cancel{x}}{a}$

(143) solⁿ

$-5x + 20 < -15$

$-5x < 35$

$x > 7 \Rightarrow \boxed{x \in (7, \infty)}$

option - (a)

---

(144) solⁿ

Sum of coefficient ~~of~~ in expension of $(1-x)^{10}$

put $x = 1$

then $= (1-1)^{10} = 0^{10} = 0$ Ans

option - (b)

---

(145) solⁿ expension of $\frac{1}{\sqrt{6-3x}}$

$= (6-3x)^{-1/2}$

$= 6^{-1/2}\left(1 - \frac{3}{6}x\right)^{-1/2}$

$= \frac{1}{\sqrt{6}}\left(1 - \frac{x}{2}\right)^{-1/2}$

for expension $\left|-\frac{x}{2}\right| < 1$

$\boxed{|x| < 2}$ Ans option - (b)

---

(146) solⁿ

p^{th} term from the end is ~~equ~~ $(n-p+2)^{th}$ term from begining

5^{th} term from the end $= (12-5+2)^{th}$ term from begining

$= 9^{th}$

$T_9 = {}^{12}C_8 \left(\frac{x^3}{2}\right)^4 \left(\frac{-2}{x^2}\right)^8$

$= \boxed{\frac{7920}{x^4}}$ option - (a)

(147) $\underline{\text{sol}^n}$: Independent term of x in expansion of $\left(\frac{3}{2}x^2 - \frac{1}{3x}\right)^9$

$$T_{r+1} = {}^9C_r \left(\frac{3}{2}x^2\right)^{9-r} \left(-\frac{1}{3x}\right)^{r}$$

$$= {}^9C_r \left(\frac{3}{2}\right)^{9-r} \cdot x^{18-2r} \times \left(-\frac{1}{3}\right)^r \cdot \frac{1}{x^r}$$

$$= {}^9C_r \left(\frac{3}{2}\right)^{9-r} \left(-\frac{1}{3}\right)^r x^{18-3r}$$

for Independent from x then $18 - 3r = 0$

$r = 6$

Now put $r = 6$

$$T_7 = {}^9C_6 \left(\frac{3}{2}\right)^{9-6} \cdot \left(-\frac{1}{3}\right)^6$$

$$= \frac{\cancel{9}\times\cancel{8}\times 7}{3\times 2} \times \frac{\cancel{3}\times\cancel{3}\times\cancel{3}}{\cancel{2}\times\cancel{2}\times\cancel{2}} \times \frac{1}{\cancel{3}\times\cancel{3}\times 3\times\cancel{3}\times\cancel{3}\times\cancel{3}}$$

$$= \frac{7}{18}$$ Ans-

Option - (a)

---

(148) $\underline{\text{sol}^n}$ Let $1, G_1, G_2, 64$

$ar^{4-1} = 64$

$r^3 = 64 \Rightarrow r = 4$

$\underline{G_1 = 4, \ G_2 = 16}$

Option - (c)

---

(149) $\left(x + \frac{1}{x}\right)^{10}$

middle term $= \left(\frac{n}{2}+1\right)^{th}$ term

$= \left(\frac{10}{2}+1\right)^{th} = 6^{th}$

$$T_6 = {}^{10}C_5 \, x^5 \cdot \left(\frac{1}{x}\right)^5$$

$$\boxed{T_6 = {}^{10}C_5}$$ Ans

Option - (b)

150 n^{th} term of 3, 10, 17 --- $= 3+(n-1)7$

and

n^{th} term of 63, 65, 67 ---- $= 63+(n-1)2$

Here given both n^{th} term are equal

then

$$3+(n-1)7 = 63+(n-1)2$$

$$5(n-1) = 60$$

$$n-1 = 12$$

$$\boxed{n = 13}$$ Ans option - (a)

---

151 solⁿ If A.M, G.M and H.M are equal between two positive number a and b then

$$\boxed{a = b}$$ option - (d).

---

152 solⁿ

$x,\ 2x+2,\ 3x+3$ are in G.P then

$$\frac{2x+2}{x} = \frac{3x+3}{2x+2}$$

$$4x^2+4+8x = 3x^2+3x$$

$$x^2+5x+4=0$$

$$(x+4)(x+1)=0$$

$$x=-4 \text{ and } x=-1$$

let $x=-4$

then $r = \frac{2x+2}{x} = \frac{2\times -4+2}{-4} = 3/2$

$$T_4 = ar^{4-1} = -4\times\left(\frac{3}{2}\right)^3 = \boxed{-\frac{27}{2}}$$ Ans option - (d)

(153) $\underline{\text{Sol}^n}$ if x, y, z are in AP then $(x+y-z)(y+z-x) = ?$

Let $x=1, y=2, z=3$ are in ~~be~~ A.P

$= (x+y-z)(y+z-x) = (1+2-3)(2+3-1) = 0$

Now put value of x, y, z in given option and find out which option gives as zero

by option (1) $8yz - 3y^2 - 4z^2$

$8 \times 2 \times 3 - 3 \times 4 - 4 \times 9$

$= 48 - 12 - 36 = 0$

$\Rightarrow \boxed{(x+y-z)(y+z-x) = 8yz - 3y^2 - 4z^2}$ Ans

option (a)

---

(154) $\underline{\text{Sol}^n}$

A rational Number in its lowest terms can be expressed as a terminating decimal iff the denominator has no prime factor other then $\underline{2 \text{ and } 5}$

because All Number ~~devis~~ completly devisible ~~by~~ or terminating decimal, If there are in denominator as $\underline{\underline{2 \times 5}} = 10$

Option – (3) $\underline{\underline{a}}$

(155) Each prime number has only two factors as 1 and its self

Option- (c) Ans

---

(156) sol^n

$ix^2 - 2(i+1)x + (2-i) = 0$

if two roots are α and β

given $\alpha = 2-i$ then $\beta = ?$

Multiple of roots $= \alpha\beta$

$$\alpha\beta = c/a$$

$$(2-i)\times\beta = \frac{2-i}{i}$$

$$\beta = \frac{\cancel{(2-i)}}{i\,\cancel{(2-i)}} = \frac{1}{i} = -i$$ Ans

Option - (a)

---

(157) if a and b are odd Integers

$2ax^2 + (2a+b)x + b = 0 \qquad a \neq 0$

$$D = b^2 - 4ac$$
$$= (2a+b)^2 - 4\times 2a\times b$$
$$= (2a+b)^2 - 8ab$$

for Any odd Integers for a, b we get $D > 0$

$\Rightarrow$ then roots are rational Option (a)

(158) α, β are the roots of $ax^2+bx+c=0$

then $\alpha+\beta = -b/a \qquad \alpha\beta = c/a$

How $\quad ax^2+2bx+4c=0$

$$x^2+2\frac{b}{a}x+4\frac{c}{a}=0$$

put value of $\frac{b}{a}$ and $\frac{c}{a}$ from above

$$x^2-2(\alpha+\beta)+4\alpha\beta=0$$

if roots are ν, δ then

$$\nu+\delta = 2(\alpha+\beta) \quad \text{—(I)} \qquad \text{and} \quad \nu\delta = 4\alpha\beta$$

$$(\nu-\delta)^2 = (\nu+\delta)^2 - 4\nu\delta$$

$$= 4(\alpha+\beta)^2 - 4\times4\alpha\beta$$

$$= 4(\alpha-\beta)^2$$

$$(\nu-\delta) = 2(\alpha-\beta) \quad \text{—(II)}$$

from solving (I) and (II) we get

$$\nu = \frac{\alpha}{2} \quad \text{and} \quad \delta = \beta/2$$

option – (a) n

(159) $2\log(x+1) - \log(x^2-1) = \log 2$

$$\log(x+1)^2 - \log(x^2-1) = \log 2$$

$$\log\frac{(x+1)^2}{(x^2-1)} = \log 2$$

$$(x+1)^2 = 2(x^2-1)$$

$$x^2-2x-3=0$$

$$(x+1)(x-3)=0 \Rightarrow x=3,-1$$

So – $\boxed{x=3}$ option – (4) or α_2

(160)

$A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$ then $A^2 - 5A = ?$

$A - \lambda I = 0$

$$\begin{bmatrix} 3-\lambda & -5 \\ -4 & 2-\lambda \end{bmatrix} = 0$$

$$(3-\lambda)(2-\lambda) - 20 = 0$$

$$\lambda^2 - 5\lambda - 14 = 0$$

Matrix A staisfy the eqn then

$$A^2 - 5A - 14I = 0$$

$$\boxed{A^2 - 5A = 14I}$$

option - (c) Ans

---

(161) $A^2 = 2A - I$

$A^n = ?$

for $n = 3$

$$A^3 = A \cdot A^2$$
$$= A(2A - I)$$
$$= 2A^2 - A$$
$$= 2(2A - I) - A$$
$$= 4A - 2I - A$$
$$= 3A - 2I \quad \text{— (i)}$$

for $n = 4$

$$A^4 = A^2 \cdot A^2$$
$$= (2A - I)(2A - I)$$
$$= 4A^2 + I - 4A$$
$$= 4(2A - I) + I - 4A$$
$$= 4A - 3I \quad \text{— (ii)}$$

from (i) and (ii) we get

$$\boxed{A^n = nA - (n-1)I}$$

option - (a) Ans

(162) Soln

$$\begin{vmatrix} b+c & a & a \\ b & c+a & b \\ c & c & a+b \end{vmatrix}$$

Let $a=1 \quad b=2, \ c=3$

$$\begin{vmatrix} 2+3 & 1 & 1 \\ 2 & 3+1 & 2 \\ 3 & 3 & 1+2 \end{vmatrix} = \begin{vmatrix} 5 & 1 & 1 \\ 2 & 4 & 2 \\ 3 & 3 & 3 \end{vmatrix}$$

$= 5(12-6) - 1(6-6) + 1(6-12)$

$= 30 - 0 - 6 \quad = 24$

$= 4 \times 1 \times 2 \times 3$

$= 4 \times a \times b \times c \quad = \underline{\underline{4abc}}$ Ans

option - (c) Ans

---

(163)

$$\begin{vmatrix} 5 & 15 & -25 \\ 7 & 21 & 30 \\ 8 & 24 & 42 \end{vmatrix}$$

$C_1 \Rightarrow 3C_1 - C_2$

$$\Delta = \begin{vmatrix} 0 & 15 & -25 \\ 0 & 21 & 30 \\ 0 & 24 & 42 \end{vmatrix} = 0 \text{ Ans}$$

Option (d) Ans

(164)

$xy - x - y + 1 = 0$

$x(y-1) - 1(y-1) = 0$

$(x-1)(y-1) = 0$

$x - 1 = 0 \quad — (\text{I})$

$y - 1 = 0 \quad — (\text{II})$

and $\quad ax + 2y - 3 = 0 \quad — (\text{III})$

from (1) and (2) put value of x and y in (3)

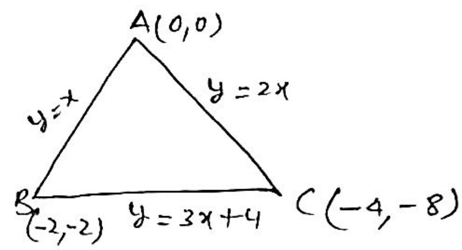
$a \times 1 + 2 \times 1 - 3 = 0$

$\boxed{a = 1}$

Option – (d)

---

(165)



$$\text{Area} = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ -2 & -2 & 1 \\ -4 & -8 & 1 \end{vmatrix}$$

$= \frac{1}{2}\left[1\,(16-8)\right] = \frac{8}{2} = 4$ squ.unit

Option (a)

---

(166)

Perpendicular distance from point α, β to the line $ax + by + c = 0$ is $\left|\dfrac{a\alpha + b\beta + c}{\sqrt{a^2 + b^2}}\right|$

Now $\perp$ distance from $(0,0)$ to the line $3x + 4y + 1 = 0$

is $= \left|\dfrac{0 \times 3 + 4 \times 0 + 1}{\sqrt{3^2 + 4^2}}\right| = \dfrac{1}{5}$

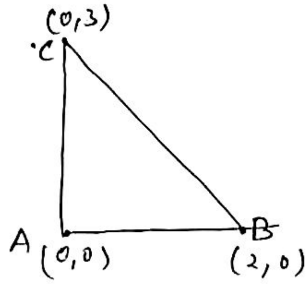
Option (d)

(167) Here

$\cancel{AB}^2 =$

$BC^2 = AB^2 + AC^2$

so Δ is Right angled Δ



Option - (b)

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(168) $\underline{\underline{Sol^n}}$

$$\tan\theta = \frac{2\sqrt{h^2 - ab}}{a+b}$$

$$= \frac{2\sqrt{\left(\frac{-7}{2}\right)^2 - 2\times 3}}{5} = \frac{2\times 5/2}{5} = 1$$

$$\tan\theta = 1 \implies \theta = 45^\circ$$

Option - (b)

---

(169) $\underline{\underline{Sol^n}}$

$$2x^2 + 2y^2 - 6x + 8y + k = 0$$

$$x^2 + y^2 - 3x + 4y + \frac{k}{2} = 0$$

for point circle radius is zero

$$= \sqrt{\left(\frac{-3}{2}\right)^2 + \left(\frac{4}{2}\right)^2 - \frac{k}{2}} = 0$$

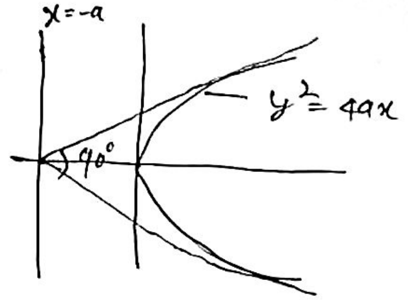
$$= \frac{9}{4} + 4 - \frac{k}{2} = 0$$

$$\frac{k}{2} = \frac{25}{4} \implies \boxed{k = \frac{25}{2}}$$

option - (c)

170

two ⊥ tangent Intersect to each other to the parabola $y^2 = 4ax$ at directric $x = -a$



$= \boxed{x + a = 0}$

option - (b)

171

$$5x^2 + 9y^2 = 45$$

$$\frac{x^2}{9} + \frac{y^2}{5} = 1$$

length of latus ractum $= = \frac{2b^2}{a}$

$$= \frac{2 \times 5}{3} = \frac{10}{3} \text{ Ans}$$

option - (a)

172

Maximum value of $\sin\theta \cdot \cos\theta$

$$= \frac{2\sin\theta \cdot \cos\theta}{2}$$

$$= \frac{\sin 2\theta}{2}$$

$= \frac{1}{2}$ (maximum value of $\sin 2\theta$ is 1

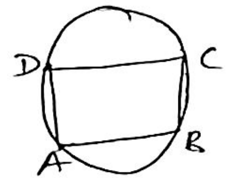
Maximum value of $\sin\theta \cdot \cos\theta = \frac{1}{2}$

Ans

option - (b)

(173) Sol

for cyclic quadrilateral



$\angle B + \angle D = 180$

$D = 180 - B$

$\cos B + \cos D = \cos B + \cos(180 - B)$

$= \cos B + (-\cos B)$

$= \cos B - \cos B$

$$\boxed{\cos B + \cos D = 0}$$

Option - (d)

---

(174) Sol^n

$3 \sin 2\theta = 2 \sin 3\theta$ and $0 < \theta < \pi$

find value $2 \sin 2\theta = ?$

$3 \sin 2\theta = 2 \sin 3\theta$

$\cancel{2} \times 3 \sin\theta \cos\theta = \cancel{2}[3\sin\theta - 4\sin^3\theta]$

$3\cos\theta = 3 - 4\sin^2\theta$

$= 3 - 4 + 4\cos^2\theta$

$4\cos^2\theta - 3\cos\theta + 1 = 0$

$(4\cos\theta + 1)(\cos\theta - 1) = 0$

$\cos\theta = 1$ and $\cos\theta = -1/4$

for $\cos\theta = -1/4$ $\quad \sin\theta = \frac{\sqrt{15}}{4}$

Now

$\sin 2\theta = 2 \sin\theta \cdot \cos\theta$

$2\sin 2\theta = 4 \sin\theta \cdot \cos\theta$

$= \cancel{4} \times \frac{\sqrt{15}}{4} \times \frac{1}{4} = \frac{\sqrt{15}}{4}$

$$\boxed{2\sin 2\theta = \frac{\sqrt{15}}{4}}$$

Option - (3)

(175) $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right) =$

$$= \frac{\pi}{3} + 2 \times \frac{\pi}{6} = \frac{2\pi}{3}$$

Option – (d)

---

(176) if $B = 2\sin^2 x - \cos 2x$ then

$$2\sin^2 x - (1 - 2\sin^2 x)$$

$$= 4\sin^2 x - 1$$

we know

$$-1 \le \sin x < 1$$

$$0 \le \sin^2 x \le 1$$

$$0 \le 4\sin^2 x \le 4$$

$$0 - 1 \le 4\sin^2 x - 1 \le 4 - 1$$

$$-1 \le 4\sin^2 x - 1 \le 3$$

$$\boxed{-1 \le B \le 3}$$

option – (a)

---

(177) ~~isin~~ if $\sin^{-1} x = \pi/5$ then $\cos^{-1} x = ?$

we know

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

$$\frac{\pi}{5} + \cos^{-1} x = \frac{\pi}{2}$$

$$\cos^{-1} x = \frac{\pi}{2} - \frac{\pi}{5}$$

$$\boxed{\cos^{-1} x = \frac{3\pi}{10}}$$

Option – (b)

(178)

$$\tan^{-1}(1+x) + \tan^{-1}(1-x) = \frac{\pi}{2}$$

$$\tan^{-1}\left(\frac{1+\cancel{x}+1-\cancel{x}}{1-(1+x)(1-x)}\right) = \frac{\pi}{2}$$

$$\tan^{-1}\left(\frac{2}{1-1+x^2}\right) = \frac{\pi}{2}$$

$$\tan^{-1}\left(\frac{2}{x^2}\right) = \frac{\pi}{2} \Rightarrow \frac{2}{x^2} = \tan\left(\frac{\pi}{2}\right)$$

$$= \frac{2}{x^2} \Rightarrow \infty$$

$$\Rightarrow x^2 = 0$$

$$\Rightarrow \boxed{x = 0}$$ option (c) a

---

(179) sol^n $A = \sin\left[\frac{1}{2}\cot^{-1}\left(-\frac{3}{4}\right)\right]$

$$A = \sin\left[\frac{1}{2}\left(\frac{\pi}{2} - \tan^{-1}\left(-\frac{3}{4}\right)\right)\right]$$

$$A = \sin\left[\frac{\pi}{4} - \frac{\tan^{-1}(-3/4)}{2}\right]$$ put $\frac{\tan^{-1}(-3/4)}{2} = t$

$$A = \sin\left[\frac{\pi}{4} - t\right]$$

$$= \sin\frac{\pi}{4}\cdot\cos t - \sin t\cos\frac{\pi}{4}$$

$$A = \frac{1}{\sqrt{2}}(\cos t - \sin t)$$

$$A^2 = \frac{1}{2}\left[1 - \sin 2t\right]$$

$$= \frac{1}{2}\left[1 - \sin \cancel{2} \times \frac{\tan^{-1}(-3/4)}{\cancel{2}}\right]$$ put value of t

$$= \frac{1}{2}\left[1 - \sin\left\{\tan^{-1}(-3/4)\right\}\right] = \frac{1}{2}\left[1 - \sin\left[\sin^{-1}\left(\frac{3}{5}\right)\right]\right]$$

$$A^2 = \frac{1}{2}\left[1 - \frac{3}{5}\right] \Rightarrow A^2 = \frac{1}{2}\times\frac{2}{5} = \boxed{A = \frac{1}{\sqrt{5}}}$$ option (a)

(180)

$$Y = \sqrt{y + \sin x}$$

then square both side

$$y^2 = y + \sin x$$

$$2y\frac{dy}{dx} = \frac{dy}{dx} + \cos x$$

$$\boxed{\frac{dy}{dx} = \frac{\cos x}{2y-1}}$$

option — (c)

---

(181)

$$x = \sin^{-1}(3t - 4t^3) \qquad y = \cos^{-1}\sqrt{1-t^2}$$

put $t = \sin\theta \Rightarrow \theta = \sin^{-1} t$

$$x = \sin^{-1}(3\sin\theta - 4\sin^3\theta) \qquad y = \cos^{-1}\sqrt{1-\sin^2\theta}$$

$$= \sin^{-1}(\sin 3\theta) \qquad = \cos^{-1}(\cos\theta)$$

$$= 3\theta \qquad y = \theta$$

$$= 3\sin^{-1} t \qquad y = \sin^{-1} t$$

$$\frac{dx}{dt} = \frac{3}{\sqrt{1-t^2}} \qquad \frac{dy}{dt} = \frac{1}{\sqrt{1-t^2}}$$

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1/\sqrt{1-t^2}}{3/\sqrt{1-t^2}} = \frac{1}{3}$$

$$\boxed{\frac{dy}{dx} = \frac{1}{3}}$$

option — (d)

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(182)

$$\lim_{x\to\infty} \frac{\sqrt{x^2-1}}{2x+1}$$

$$\lim_{x\to\infty} \frac{\cancel{x}\sqrt{1-\frac{1}{x^2}}}{\cancel{x}(2+\frac{1}{x})} = \frac{\sqrt{1-0}}{2+0} = \frac{1}{2} \text{ Ans}$$

option — (d)

(183) $\frac{d}{dx}(\cos^{-1}x) = -\frac{1}{\sqrt{1-x^2}}$

is valid only for $-1 < x < 1$

option - (a)

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(184) $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$

put $x = \tan\theta$

$\tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$

$\tan^{-1}\left(\frac{\sec\theta - 1}{\tan\theta}\right)$

$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$

$= \tan^{-1}(\tan\theta/2)$

$= \frac{\theta}{2} = \frac{1}{2}\tan^{-1}x$

Now derivative of $\frac{1}{2}\tan^{-1}x$ with respect to $\tan^{-1}x$ is

$$= \frac{\frac{1}{2}\times\frac{1}{\sqrt{1-x^2}}}{\frac{1}{\sqrt{1-x^2}}} = \frac{1}{2}.$$

option — (3)

B

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(185) $f(x) = \frac{1}{x+1} - \log(1+x)$

$f'(x) = -\frac{1}{(x+1)^2} - \frac{1}{1+x}$

$= \frac{-1-(1+x)}{(x+1)^2} = \frac{-(2+x)}{(x+1)^2} = \frac{-(x+2)}{(x+1)^2}$

$\Rightarrow f'(x) < 0$

$\Rightarrow f(x)$ is decreasing function option (b)

186 Maximum value of $f(x) = \frac{x}{4+x+x^2}$ on $[-1, 1]$ is

$$f'(x) = \frac{(4+x+x^2)1 - (0+1+2x)x}{(x^2+x+4)^2}$$

$$= \frac{x^2 + x + 4 - \cancel{x} - 2x^2}{(x^2+4+x)^2}$$

$$= \frac{4-x^2}{(x^2+x+4)^2}$$

for ~~point of~~ maxima/minima $f'(x) = 0$

$$\frac{4-x^2}{(x^2+x+4)^2} = 0$$

$$(2-x)(2+x) = 0$$

$x = 2, -2$ which Not lie in $[-1, 1]$

So point of maxima is 1

maximum value $= \frac{1}{4+1+1} = \frac{1}{6}$ Ans

option – (c) Ans

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187 Angle of tangent to the parabola

$x^2 = 2y$ at $(1, \frac{1}{2})$

$2x = 2\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = x$

$\Rightarrow \left(\frac{dy}{dx}\right)_{(1, \frac{1}{2})} = x \Rightarrow \tan\theta = 1$

$=$

$\boxed{\theta = \pi/4}$

option – (b)

188 Normal is $\parallel$ to x — axis

Mean eqn of Normal $y = K$

slope of normal $= \frac{dy}{dx} = 0$

~~slope of tangent~~ $\cancel{= \frac{1}{\frac{dy}{dx}}} = \infty$

for the curve $\Rightarrow \frac{dx}{dy} = 0$

option — d

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189 soln The largest Interval for which

$x^{12} - x^9 + x^4 - x + 1 > 0$

the largest Interval is $\boxed{-\infty < x < \infty}$

option — d

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190 $\int \frac{dx}{e^x - 1} = \int \frac{dx}{e^x(1 - \frac{1}{e^x})}$

$= \int \frac{e^{-x}\,dx}{1 - e^{-x}}$

$= \underline{\log|1 - e^{-x}| + c}$ option — c

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191 $\int \frac{dx}{x\sqrt{1 - x^3}}\,dx$

put $x^3 = \sin\theta$

$x = \sin^{1/3}\theta$

$dx = \frac{1}{3}\sin^{-2/3}\theta\,\cos\theta\,d\theta$

$= \int \frac{1}{3\sin^{2/3}} \cdot \frac{\cos\theta\,d\theta}{\sin^{1/3}\theta \cdot \sqrt{1 - \sin\theta}} = \frac{1}{3}\int \frac{\cos\theta\,d\theta}{\sin\theta \cdot \sqrt{1 - \sin\theta}}$

(191) Again put $1-\sin\theta = t^2 \Rightarrow \sin\theta = 1-t^2$

$\therefore -\cos\theta\, d\theta = 2t\,dt$

$\cos\theta\, d\theta = -2t\,dt$

Now $\frac{1}{3}\int \frac{-2\cancel{t}\,dt}{(1-t^2)\cancel{t}} = \frac{1}{3}\int \frac{-2\,dt}{1-t^2} = \frac{2}{3}\int \frac{dt}{t^2-1}$

$= \frac{\cancel{2}}{3} \times \frac{1}{\cancel{2}} \log\left(\frac{t-1}{t+1}\right)$

$= \frac{1}{3}\log\left(\frac{\sqrt{1-\sin\theta}-1}{\sqrt{1+\sin\theta}+1}\right) \Rightarrow \frac{1}{3}\log\left(\frac{\sqrt{1-x^3}-1}{\sqrt{1+x^3}+1}\right)$ Ans

Option – (a)

(192) $\int_2^3 \frac{dx}{x^2-x} = \int_2^3 \frac{dx}{x(x-1)} = \int_2^3 \left(\frac{1}{x-1} - \frac{1}{x}\right)dx$

$= \left[\log(x-1) - \log x\right]_2^3$

$= \log\left(\frac{x-1}{x}\right] = \log\left(\frac{3-1}{3}\right) - \log\left(\frac{2-1}{2}\right)$

$= \log\left(\frac{2}{3}\right) - \log\left(\frac{1}{2}\right)$

$= \log \frac{2}{3} \times \frac{2}{1}$

$= \log\left(\frac{4}{3}\right)$ Ans

Option – (c) Ans

(193) if $\int e^x\left[f(x) + f'(x)\right]dx = e^x \sin x$

we know $\int e^x\left[f(x) + f'(x)\right] = e^x f(x)$

$\Rightarrow \boxed{f(x) = \sin x}$

Option ⇒ (b)

(194)

$$2x\frac{dy}{dx} - y = 3$$

$$\frac{dy}{dx} = \frac{y+3}{2x}$$

$$\frac{dy}{y+3} = \frac{dx}{2x}$$

$$\log(3+y) = \frac{1}{2}\log x + \log c$$

$$y+3 = c\sqrt{x}$$

$$(y+3)^2 = xc^2$$

$(y+3)^2 = Kx$ like as parabola $y^2 = 4ax$

= Option — (c)

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(195)

$$y = A\cos\alpha x + B\sin\alpha x$$

$$\frac{dy}{dx} = -A\alpha\sin\alpha x + \alpha B\cos\alpha x$$

$$\frac{d^2y}{dx^2} = -A\alpha^2\cos\alpha x - \alpha^2 B\sin\alpha x$$

$$= -\alpha^2[A\cos\alpha x + B\sin\alpha x]$$

$$= -\alpha^2 y$$

$$\boxed{\frac{d^2y}{dx^2} + \alpha^2 y = 0}$$

Option — (b)

(196) Area b/w $y^2 = 4ax$ and line $y = mx + c$ is $\frac{8}{3}\frac{a^2}{m^3}$

For curve $y^2 = x$ and $2y = x$

$a = \frac{1}{4}$ $m = 1/2$

Area $= \frac{8}{3}\frac{a^2}{m^3} = \frac{\cancel{8}^{4}}{3} \times \frac{1}{\cancel{16}_{2}} \times \frac{\cancel{8}}{1} = \frac{4}{3}$ sq unit

Option (a)

---

(197)

$$\frac{dy}{dx} = \left(\frac{y}{x}\right)^{1/3}$$

$$\frac{dy}{y^{1/3}} = \frac{dx}{x^{1/3}}$$

$$y^{-1/3}\,dy = x^{-1/3}\,dx$$

Now Integration both side.

$$\frac{y^{2/3}}{2/3} = \frac{x^{2/3}}{2/3} + K$$

$$y^{2/3} - x^{2/3} = \frac{2}{3}K$$

$$\boxed{y^{2/3} - x^{2/3} = C}$$

---

(198) I.f of $\frac{dy}{dx} + \frac{y}{x} = x^2 - 3$

$$\text{I.f} = e^{\int \frac{1}{x}dx}$$

$$= e^{(\log_e x)} = x$$

Option - (C)

199 Solⁿ $\hat{i}+3\hat{j}-2\hat{k}$, $2\hat{i}-\hat{j}+4\hat{k}$ and $3\hat{i}+2\hat{j}+x\hat{k}$ are coplaner

then

$$\begin{vmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 3 & 2 & x \end{vmatrix} = 0$$

$$1(-x-8)-3(2x-12)-2(4+3)=0$$

$$-x-8-6x+36-14=0$$

$$-7x=-14$$

$$7x=14$$

$$\boxed{x=2}$$

Option - (d)

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200 Solⁿ $\bar{p}=3\bar{a}-5\bar{b}$ $\quad \bar{q}=\bar{a}+\bar{b}$

$\bar{p}$ and $\bar{q}$ are $\perp$ then

$$\bar{p}\cdot\bar{q}=0$$

$$(3\bar{a}-5\bar{b})(\bar{a}+\bar{b})=0$$

$$3\bar{a}\cdot\bar{a}+3\bar{a}\cdot\bar{b}-5\bar{b}\cdot\bar{a}-5\bar{b}\cdot\bar{b}=0$$

$$3+2a\cdot b-5=0$$

$$a\cdot b=-1$$

$$|a|\cdot|b|\cos\theta=-1$$

$$\cos\theta=-1$$

$$\theta=\pi$$

$\Rightarrow$ $\bar{a}$ and $\bar{b}$ have opposite direction

Option - (b)